# Fusing foundations: How similar are foundational debates in mathematics and science?

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## Introduction

In this talk I want to present themes from my research in the **foundations of mathematics**.

My interest though is **similarities with the philosophy of science** more broadly.

I'll **suppress** mathematical detail for clarity (though please ask if you're curious).

Rather than bombarding you with arguments, I want to just introduce some ideas to **generate discussion** with the following background question:

> **Underlying question.** How similar are (or what are the similarities between) foundational debates in mathematics and science?

So—let's get going!

## 1 Justification and fallibilism

Often, mathematics is seen as **distinctive**, in that justification is **infallible**.

e.g. Because (i) justification is **proof from the axioms**, (ii) the axioms are **true**, and (iii) **proof** preserves **truth**.

I've challenged this though with **Mathematical Gettier Cases** (draft available if you're interested).

One idea there is (drawing on work of Silvia De Toffoli) that the (a?) notion of justification is not really proof but something **weaker** (simil-proof). Let's set this aside (but happy to chat in discussion!).

A different place we might get fallibilism is in the **justification of axioms**.

How do we know we get the **true axioms**?

We might be **lucky** in that we prove a **true proposition** from **false 'axioms'**?

This is **especially deep** in virtue of the following:

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**Independence Phenomenon.** There are many sentences for which it is (provably) neither the case that ZFC proves $\phi$ nor that ZFC proves $\neg\phi$.

There are **a lot** of sentences of this form.

So should we **add** to our axiom system? If so **how**?

In my opinion this will involve **complex stories** weaving together **multiple considerations**.

**One** aspect of such a story might be the following: There's something like **prediction**, **confirmation**, and **accommodation** with respect to set-theoretic axioms.

Here's Gödel:

> Furthermore, however, even disregarding the intrinsic necessity of some new axiom, and even in case it had no intrinsic necessity at all, a decision about its truth is possible also in another way, namely, **inductively by studying its "success"**, that is, its **fruitfulness** in consequences and in particular in "**verifiable**" consequences i.e., **consequences demonstrable without the new axiom**, whose proofs by means of the new axiom, however, are **considerably simpler and easier to discover**, and make it possible to **condense into one proof many different proofs**. (Gödel, 1947 version of 'What is Cantor's continuum problem?', p. 182)

There's **so** much going on in this quotation. Some questions:

(1.1) What are these **mathematical data** (and how are they **similar to** and **different from** other kinds of data)?

Our suggestion (from 'On Forms of Justification in Set Theory') they are the **currently accepted mathematical truths**.

(1.2) What is meant by **fruitfulness**?

I think Gödel has this **verifiability** idea in mind, but others (e.g. Penelope Maddy) have taken this idea of fruitfulness in a **different** direction (fruitful consequences are **mathematically deep** consequences).

(1.2) Are there differences (as there often are in the philosophy of science) between **prediction**, **accommodation**, and the level of **surprise** involved?

(1.3) In what sense are these methods **inductive**? Are they more **abductive** in nature?

(1.4) And how do they relate to the existence of an **underlying conception** for the relevant subject matter?

## 2 Adopting conceptions

I just spoke about **conceptions**. But what are they?

Loosely speaking, they're descriptions of what the sets are **like** in order to motivate a **theory**.

**Example.** The *iterative conception* holds that sets are formed in **stages**, starting with the empty set, forming all possible subsets at successor stages, and collecting unions at limits.

The iterative conception is **often** thought to motivate ZFC.

The iterative conception is often thought to solve problems like **Russell's Paradox**.

There's a kind of story on which we started working **naively** with set theory (around the turn of the $20^{\text{th}}$ century), discovered it was **inconsistent** (via Russell's Paradox), and then realised that the iterative conception is **correct** (roughly in the period 1900–1970) and then lived **happily ever after**.

But this just isn't right (an idea explored by lots of people), the conceptual history of set theory is one with **many** twists and turns and **conceptual forks**.

It's unclear that the iterative conception **had** to be the one we chose.

In a book (*Engineering Set-Theoretic Concepts*) I'm working on (e-mail for a draft!) I suggest that we're at a possible conceptual choice point **now** (and I draw some links to literature on **conceptual engineering**).

There's a number of questions that are raised:

(2.1) Should we be **pluralists** here? Is there a **disanalogy** between pluralism in the foundations of mathematics and science?

(2.2) What are the **constraints** governing choice of conception? And **can** we even '**choose**' (cf. Cappelen)?

(2.3) What of **open texture**? e.g. Wilson's plane, Shapiro and computation, set theory (both past and present).

## 3 External perspectives

The final theme I want mention involves the use of **extensions**.

Here's one natural view:

> **Universism.** There is a unique maximal universe of sets that contains every possible set.

But there's lots of constructions in set theory that **add** sets to models (e.g. forcing).

This has led to:

> **Multiversism.** There is no one maximal universe of sets, any universe of sets can have new sets added to it.

One interesting fact about set theory is the following (see 'Forcing and the Universe of Sets' and 'Universism and Extensions of $V$'):

> **Use of extensions.** You can use extensions of the/a universe in **formulating axioms** and **proving theorems** about the/a universe.

One can then formulate a version of the **indispensability argument** for extensions (though Quine wouldn't have liked this):

**Premise 1.** We should only accept the existence of entities that are **indispensable** for our best scientific theories (including set theory).

**Premise 2. Extensions are indispensable** to our best set theories.

**Conclusion.** We should accept the **existence** of extensions of any universe.

**Corollary.** Universism is **false**.

This gets quite mathematically complex (in particular there are versions of Field-style **hard road nominalism** that I've explored).

I want to close with the following **question**:

**Question.** Are there **similar** kinds of situation in the sciences?

Of course we can learn about (a mathematical model of) the world by embedding it in some **larger** (mathematical) space.

But are there cases where we think we learn about the physical universe on the basis of viewing it as embedded in some **larger physical** structure.

One suggestion: This could be how one thinks of the (physical) **multiverse** (in addition to being a response to fine-tuning arguments).

As mentioned above, there are debates about **set-theoretic** multiverses too.

There's a language use question about how apt this term borrowed from physics is for the set-theoretic case, but it is underpinned by more **substantial** questions surrounding the similarities and differences between these two fields.

# Takk for at du lyttet!