Week 8. Mathematical Intuition

Dr. Neil Barton
n.a.barton@ifikk.uio.no
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Recap

Last week we considered **nominalism**: The idea that mathematical objects don't exist (as abstract entities).

Part of what this view attempted to do was deal with was the:

Epistemological Challenge. How can we gain knowledge of mathematical objects if they are acausal etc.?

Some nominalists (e.g. Field) attempted to deal with this by reducing talk of mathematical entities to talk of physical entities.

This week we'll see accounts that seek to maintain platonism and provide an epistemology in those terms.

1 Roles of intuition

Some kind of intuition has been around for a long time.

For example, Descartes' 'clear and distinct' perception might be an example of this, and certainly the idea goes back further (e.g. Plato).

We'll consider:

Mathematical intuition Mathematical entities can be known through *something like* perceptual means, even if we're platonists.

Note: 'Intuition' can mean a lot of things like 'feel that something is the case'. *Mostly* people have in mind something more robust in this context.

> But, despite their remoteness from sense-experience, we do have something like a perception of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e. m mathematical intuition, than in sense-perception. (Gödel, 'What is Cantor's Continuum Problem?', 1964)

Note: Gödel's views are very open to many interpretations (and in particular his notebooks are very instructive here).

The idea here is that we have something like a faculty of perception, but one that can be directed to the mathematical rather than the physical realm.

Let's distinguish between:

Propositional perception. I perceive that ϕ (or have a perception as of ϕ).

This further subdivides into perception of **axioms** and perception of something **following** from the axioms.

e.g.1. I perceive that the Axiom of Extensionality holds.

e.g.2. I perceive that every set has a singleton (from the axioms of ZFC).

e.g.3. I perceive that the Ordinal Inner Model Hypothesis implies that $0^\sharp$ exists. (This could be whatever, the point is that it's a theorem I found hard.)

Note: It's a very good question as to where the line is here.

Object perception. There is then the related idea of perception **of** the objects.

e.g. I perceive (have perceptual access to) the singleton of the empty set.

Questions?

2 Points in favour of the role of mathematical intuition

There's some reasons to find this attractive:

1. You can maintain platonism with all its semantic simplicity, if you can make the account of perception fly.

2. It *seems* to deal with some worries people think holism has.

For example: Why does it seem that 2+3=5 is really *obviously true*?

Holism: It's really really really close to the centre of our web of belief (though in principle revisable).

Current view: It's true, and you can perceive it directly.

3. Something like intuition *does* seem to play a role in mathematical practice.

e.g.1. Rav, 'Why Do We Prove Theorems?'

e.g.2. The topologist's coffee cup.

e.g.3. A lot of work in the philosophy of mathematical practice (e.g. Silvia De Toffoli, Marcus Giaquinto).

Note: The import of this is very controversial.

Questions?

3 Some problems

Core questions. How should we understand this faculty of 'perception/intuition'? What is it like? How does it interact with our neurobiology?

If we think of this faculty of perception exactly in line with perception of physical objects (e.g. vision), we run into some problems.

Note: I don't want to foist this interpretation on Gödel. Note *exactly* what he says in the quotation (it's about *less confidence in the kind* of perception, rather than them being *the same*).

Problem. This perception seems entirely mysterious and unscientific.

For perception of physical objects, we seem to have at least partial explanations of how this is to work (or at least confidence that we *could* get such an explanation).

For example colour: I can talk about how light hits the surface of an object, is reflected in different wavelengths according to the surface, and then hits my retina.

(Of course the story of what happens after that is a bit trickier.)

What is the comparable story meant to be for *acausal* and *non-spatiotemporally located* objects?

Problem. (Related.) Paradigmatic instances of perception seem different from mathematical ones.

It is *obvious* that I have a perception of a coffee cup (or a perception as of a coffee cup for the sceptically inclined).

Is it *obvious* I have a perception of the number seven?

Problem. Size worries.

I am but a mere bounded finite being.

It seems that to ground (some) claims about mathematics

e.g. "The integers under addition form a group"

e.g. "The reals under addition and multiplication form a field."

e.g. "It is not the case that every function $f : \mathbb{R} \to \mathbb{R}$ is everywhere differentiable"

4 Modifying the account intuition

Parsons attempts to get around this via some suggestive remarks concerning the type/token distinction and imagination.

Types are abstract properties that apply to many **tokens** which instantiate the type.

e.g. ABBA, two types, two tokens of each type.

Contention. We learn about abstract types by studying concrete tokens.

These tokens *are* subject to perception in the normal way.

e.g. I can learn about Fat Freddys Drop's 'Roady' even if it's me rather than Dallas Tamaira singing.

Problem. This doesn't get us very far.

Suggestions.

(Parsons) Imagination has a role to play here.

We can always 'imagine' one more (e.g. for infinite objects).

Imagining a token can have the same 'form' as the abstract object.

Question. Is this precise enough to deliver what we want out of mathematics?

Question. What reference does this guarantee us? e.g. Does it get us reference to the *standard* model of arithmetic?

(Carey/Dehaene/Núñez) We can add in the notion of *bootstrapping* (especially in Carey)—we slowly build up via a combination of perceptual activities, but also an understanding of social practices (e.g. counting).

Fact. Some sort of imagination has a role to play in mathematics, and it would be good to account for this.

Questions?

5 An idea I'm toying around with...

Note: I'm thinking about some material along the Parsons line.

In particular, we might think of mathematics in analogy with *representationalism* in the philosophy of mind.

We can have different representations of what may be an 'inaccessible' mathematical realm.

This has a neo-Kantian flavour to it.

But: Does it suggest a perspectivalism about mathematics and mathematical truth?

(Especially for some of the more esoteric regions of foundations?)

Questions?

6 Questions/Discussion

Question. (Julius/Birgit) Imagine a number of highly intelligent children growing up without any instruction on how to think about anything related to mathematics. Do we expect them to develope the same basic concepts for percieving mathematically relevant aspects of the external world, like quantities and shapes?

Question. (Julius/Birgit) Would answering yes to this imply something analogous to Kantian categories?

Question. (Julius/Birgit) To which extent can mathematics be thought of as an endeavour to integrate and develop concepts used in perception into a maximally expressive and consistent conceptual structure?

Question. (Emma) Is mathematical intuition direct perception of mathematical objects or something else?

Question. (Katharine) Do axioms 'force themselves on us as true'?

Question. (Haochong/Birgit/Brian) What exactly does the duck/rabbit example from Føllesdal achieve in this context?

Question. (Julius) Can 'intuition' be developed? (Analogy: Think of a chess player.)

Question. (Brian) Why might Gödel have disliked Husserl's *Logical Investigations*?

Question. (Pietro) Are idealised physical objects 'ontologically similar' to mathematical ones?

Question. (Pietro) How to handle instantiations of infinite structures?