

Week 7. Nominalism

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Recap

Last week we discussed Quine's holism.

A key part of this was the idea of *indispensability*. Two key claims:

- (1.) Some mathematics is indispensable for our best scientific theories.
- (2.) If a theory that quantifies over an object is indispensable for our best scientific theories, then we should accept the existence of such an object.

This week we'll look at nominalism, the idea that mathematical objects do not exist.

1 Motivations for nominalism

According to the usual objects-realism story, mathematics is about:

A domain of (1.) **objects**, that (2.) **abstract** (acausal, non-spatiotemporally located).

This has two pleasing features:

- (1.) It gives us a nice simple semantics; truth conditions match surface syntactic structure of utterances.
- (2.) It seems to pair well with the necessity of mathematical objects (since one might think that abstracta of this kind exist at every world).

However, one big issue is that:

Epistemological Challenge. How can we gain knowledge of mathematical objects if they are acausal etc.?

There is also a family of challenges coming from ontology.

Ontological Challenges. Much of mathematical ontology *seems*:

1. Very **strange**. One might have suspicions about abstracta in general.
2. **Excessive**. The kinds of mathematics actually used in the natural sciences are a relatively small part of the possible mathematical objects we might consider.
3. Only defined **negatively**. They're *not* spatiotemporal, they're *not* causal. So what on earth are they?

One response to these clusters of issues has been:

Nominalism. There are no mathematical objects (conceived of as abstract objects).

This has the consequence that it seems that mathematics is false.

One idea: Make the mathematical objects concrete (e.g. Pen Maddy's 1990 book *Realism in Mathematics*).

We'll consider two slightly different options.

(1.) **Deflationary** nominalism (e.g. Field, Melia).

(2.) **Fictionalism** (e.g. Leng, Yablo).

We should mention also eliminative structuralism (e.g. Hellman). We'll discuss that in a future week.

Clarifications?

2 Deflationary nominalism

One way of viewing the two nominalistic options is as different reactions to the indispensability argument.

Field's programme takes aim at the idea that the mathematics used in science is *really* dispensable.

Two step process:

(1.) Provide a nominalistic paraphrase of the mathematics used in the natural sciences.

(2.) Show that detours via 'platonistic' theories of mathematics are conservative over this nominalistic paraphrase.

We won't go into the details of (1.) but Field provides a nominalisation of the theory of Newtonian gravitation.

Geometry is handled by developing a theory of synthetic geometry in terms of 'betweenness' and 'being-an-equal-distance-from' relations, and showing that it is isomorphic to standard analytic (coordinate-based) geometry.

Quantities are handled by reference to 'canonical' physical objects.

Some problems:

Problem 1. He uses the fact that arbitrary sets of geometrical spacetime points exist.

Problem 2. There are substantial questions about what can be nominalised this way (e.g. how are you going to handle an infinite dimensional space?).

Problem 3. There is a tension with naturalism (physicists tend to just use whatever they need).

Problem 4. (The Application Problem.) *Why* is mathematics so successful?

(Allegedly platonism has a better response here—physical systems can have *structural similarity* to mathematical ones.)

Clarifications?

On to *conservativeness*.

What is a conservativeness claim?

A theory T_1 is *conservative over* T_2 iff there is no sentence in the language of T_2 that is implied by T_1 but not by T_2 .

Note. There is a semantic and a proof-theoretic reading of conservativeness.

Objection. If we can encode a Gödel sentence for the nominalistic theory N (call this G_N) then adding G_N to whatever platonistic theory we're considering (call it P) will give a platonistic theory (proof-theoretically) non-conservative over N .

Response. G_N has never played any role in science.

Response-to-response. But it *could* (consider outputs of a random number generator, e.g. a radioactive isotope).

It will never output a proof of G_N from N (assuming N is consistent¹).

Instead we might say that G_N is a *semantic consequence* of N .

But what kind of access do we have to this semantic consequence relation?

Field-style nominalism is still an ongoing field of study (though it is regarded as a very hard road to travel).

Clarifications?

3 Fictionalism

A different option: Accept indispensability but deny that it should give us *truth*.

Instead opt for *fictionalism* (e.g. Mary Leng, Stephen Yablo)

Fictionalism. Mathematical objects are (useful) fictions.

Strictly speaking mathematics is *false*.

However mathematical claims can be *true-in-the-story-of* some mathematical subject matter.

Application challenge. If we're fictionalists, how is mathematics applicable (and so successfully)?

Lots to be said here.

One kind of response: We are used to the successful use of metaphor etc. in talking about the real world.

Why can't mathematics be an especially rigorous form of this?

Question. Does this deliver the level of precision we need and accept from mathematical theories?

Question. What is the *gain* over platonistic ways of thinking? Is there really a substantial difference between fictionalism and platonism?

(Thinking through this question is a good philosophical exercise.)

Clarifications?

¹Really, this should be ω -consistent, but I suppress the detail for ease of expression.

4 Questions/Discussion

Question. (Ingvild) Does nominalism have implications for the way we do science?

Question. (Johan/Nicola/Haochong/Birgit) What do we mean by 'exist' here?

Question. (Birgit/Julius/Michel) What counts as a successful science?

Question. (Katharine) What does appealing to natural science buy us beyond appealing to mathematics directly?

Question. (Pietro/Jens/Brian) What role does *impracticality* of use have for assessing theories?

Question. (Nicola) Are there similar moves going on with respect to nominalism and other areas?

Question. Does fictionalism really have such an advantage over platonism?