Week 1. Mathematics as a philosophical problem

Dr. Neil Barton
n.a.barton@ifikk.uio.no
Philosophical Logic and Mathematical Philosophy

20 January 2022

General structure of the seminars

-Generally speaking, I will talk a little at the start of each session, breaking for questions.

-We'll then take a short (5-10 minute) break.

-I will then hand over to one (or more) of the course participants to introduce some questions and thoughts for discussion.

-(This is a requirement for those who want to take the course for assessment.)

-We'll potentially split down into smaller groups for the discussion and report back (not today though, we'll keep things simple for now).

-The *aim* is to make this a *hybrid* course.

-I hope to start meeting in person (with an online option for those wishing to join remotely).

-This will depend a little on how things go with the pandemic and your preferences.

-Today I'll introduce the course, and talk about some key themes that emerge already in the work of Plato and Kant.

-We can then talk about any administrative details in the last 45 minutes or so.

-First, I'd like to hear a sentence (or two) from each of you about your background and interests.

Introduction

-Maybe you just find philosophical questions about mathematics inherently fascinating.

-If so, great! (You have chosen to come to this course after all...)

-But if not, we can get some motivation for studying mathematics philosophically.

-Mathematics is really important for understanding the world.

-Trying to figure out some philosophical underpinnings for mathematical ideas has resulted in very *useful* concepts.

-For example: Russell and Whitehead (in *Principia Mathematica*) tried to find an underpinning of mathematics in terms of logic, to do so they used something called *type theory*, which in turn is important for the λ-calculus, which then finds its way into computer science.

-By thinking about these difficult and abstract philosophical questions, sometimes we develop ideas that are really *useful* too.

-Today we'll look at two key authors (Plato and Kant) and identify some challenges that will occupy us in the rest of of the course.

1 Plato and the a priori

-The *Meno* 80a to 86c occurs in the context of Meno trying to give an account of what *virtue* is.

-Socrates bamboozles him (asking him to not give examples, but rather provide a good definition of virtue).

-Meno then presents a 'paradox' concerning knowledge acquisition.

Socrates reformulates it thus (at 80e):

"[Someone] cannot search for what he knows—since he knows it, there is no need to search—nor for what he does not know, for he does not know what to look for."

-(Interpreting this puzzle can be quite subtle.)[1]

-Socrates' response: We are not *learning*, but rather *recollecting innate ideas* via our immortal soul.

-To make this vivid, he asks a slave boy to solve some geometry problems concerning squares and their diagonals.

-At each stage (Socrates claims) the boy is not learning from him, but rather recollecting.

-OK: So Plato/Socrates' account of knowledge is very *weird*.

-But in the case of mathematics it shows the following interesting observations.

Observation 1. It seems like we often come up with mathematical *concepts* independent of any experience of the external world.

(e.g. the initial development of complex numbers.)

[1] e.g. See M.M. McCabe's 'Escaping One's Own Notice Knowing: Meno's Paradox Again'.

Observation 2. It seems like we can learn and gain evidence for propositions purely by thinking about them without needing to make observations.

-We say that mathematics is *a priori* (as opposed to *a posteriori*).

Questions?

2 Kant, the analytic/synthetic distinction, and necessary truths

-Kant (especially in the introduction to the *Critique of Pure Reason*) also picks up on the a priority of mathematical knowledge.

-(He especially highlights the difference between empirical and mathematical knowledge.)

-He also argues for some other features of mathematical knowledge.

-The first is the *analytic/synthetic* distinction.

-Kant specifically considers subject-predicate claims, and says that a statement is *analytic* if the predicate concept is contained in the subject concept and *synthetic* otherwise.

-(The example he gives is of a body being extended.)

-More generally, we can think of the analytic statements as those that are true in virtue of the *meanings* of the terms involved.

-Kant holds that the truths of mathematics are *synthetic*.

-There is nothing in the natures of 7, 5, and 12 (so Kant says) that means that $7 + 5 = 12$, we must rather bring together (say) a visualisation of 7 dots and a visualisation of 5 dots together in thought.

Questions?

-Another aspect of mathematical thought that Kant identifies is that mathematical truths seem to be *necessary*.

Necessity. Mathematical truths hold (or fail) in *all possible worlds*.

-Suppose I missed my train and couldn't teach today.

-Two plus two would still be four, and the angles of a triangle would still add up to 180 degrees (in Euclidean space).

Questions?

-Closely linked to this idea of necessity is the idea that our mathematical knowledge is especially *reliable* or *secure*.

-Once I've seen that $7 + 5 = 12$, it will take a lot to shake my confidence in this proposition.

-Moreover, mathematics can be *incredibly successfully* applied in physics etc.

-However, mathematicians do make *mistakes*.

-This crops up in the Meno (the slave boy repeatedly gets things wrong).

-But this also happens with brilliant mathematicians (e.g. Wiles' first proof of the Fermat Conjecture that $a^n + b^n = c^n$ is impossible where a, b, c are positive integers and $n > 2$).

-Vladimir Voevodsky (a winner of the Fields medal) relates the following concerning a mistake discovered in his own work.

"Starting from 1993, multiple groups of mathematicians studied my paper at seminars and used it in their work and none of them noticed the mistake."[2]

Questions?

3 Summing up

Mathematics appears to be:

1. A priori.
2. A domain of necessary truths.
3. An especially secure form of knowledge.

However, mathematical objects also appear to be *abstract*.[3]

Abstract. Mathematical objects are not *spatiotemporally located*, and they do not engage in *causal* interactions with us.

-e.g. If I draw a figure on the board (e.g. a triangle) it just isn't identical with the thing I'm talking about (e.g. the lines have width, will be wobbly etc.).

This raises the following challenge:[4]

Epistemological Challenge. How do we gain *knowledge* of mathematical objects if they are *abstract*?

4 Questions.

Question 1. Is mathematics a body of *truths*?

Question 2. Are mathematical objects/entities *abstract*?

Question 3. Is mathematics all a priori?

[2]See 'The Origins and Motivations of Univalent Foundations'. Rav's 'Why Do We Prove Theorems?' is also interesting. If you're interested, I have a paper (entitled 'Mathematical Gettier Cases and Their Implications') on the subject.

[3]This idea seems to appear already in Plato, see especially Book VII of the *Republic*.

[4]There's a large literature on this. A seminal work is Benacerraf's 'Mathematical Truth'.

Question 4. Where do our mathematical concepts come from?

Question 5. Is mathematics analytic or synthetic (and is this distinction any good)?

Question 6. How should we answer the epistemological challenge?

Question 7. How different is mathematics from other areas of knowledge? (and *is* it especially secure/necessary?)