Course Outline: Introduction to the Philosophy of Mathematics

Dr. Neil Barton
neil.barton@uni-konstanz.de

Sprechstunde: Nach Vereinbarung

1 Course description and objectives

1.1 Course description

Mathematics is an essential part of our reasoning about the world. It allows us to represent various aspects of reality, enabling us to design flying aircraft, build structures supporting many tonnes, and create small wallet-sized devices for communication, listening to music, and watching videos on YouTube (as well as displaying PDFs of course outlines).

In this course we'll examine how we should understand human mathematical endeavour. The material will be oriented around a central theme that has intrigued philosophers and mathematicians for quite some time; the idea of infinity. We begin with a discussion of the differences in attitude between Plato, the Presocratics, and Aristotle on the nature of infinity. We'll then consider how ideas gradually changed and attitudes to mathematics shifted with the development through the early modern period, in particular looking at the problems faced by Newton and Leibniz. We'll then examine one of the most exciting and tumultuous periods of the modern scientific era; the attempts to interpret mathematics of the logicist, intuitionist, and finitist positions and their disagreements and difficulties. In the last part of the course, we'll include an informal discussion of the limitative results (including Gödel's Theorems) and what remains of these programmes for the philosophy of mathematics today.

1.2 Prerequisites

A first course in logic (or knowledge of the basics of propositional and predicate logic) will greatly help students follow discussion. Students from outside philosophy are very welcome to take the course.

1.3 Objectives

By the end of this course you should be able to:

(i) Understand and use central concepts in the philosophy of mathematics.

(ii) Use these concepts to outline various important positions.

(iii) Show familiarity with texts in the philosophy of mathematics.

(iv) Evaluate these texts, positions, and ideas.

(v) Construct your own rigorous arguments for specific positions.

2 Administrative issues

2.1 Temporal and spatial location

Time: Mittwochs, 11:45–13:15

Room: C 426

Sprechstunde: Nach Vereinbarung.

2.2 E-Learning

ILIAS-Platform: `https://ilias.uni-konstanz.de` (password: aleph).

2.3 Assessment

Proseminarschein 4 ECTS:

- 80% One paper of 1750–2250 words.
- 10% attendance/contribution in class. In order to get this extra 10%, you can miss at most two seminars (but it is otherwise ungraded).

- 10% questions (three questions on the text per week to be submitted through ILIAS). Again, you can miss at most two weeks of questions to get this extra credit, but the questions themselves are ungraded.

Hausarbeit 4 ECTS

- 90% One paper of 3500–4500 words
- 10% contribution and attendance (again, two possible missed clases are allowed for the extra credit).

BA students can be assessed through either method inclusive (i.e. you can do both). Lehramtsstudierende can do *either* Proseminarschein or Hausarbeit but *not* both.

2.4 Important deadlines

Weekly: (Proseminarschein only) Deadline for questions: 23:55, Monday before the seminar.

15. January 2020: Deadline for selection of assessment in StudIS and last opportunity to cancel registered assessments.

29. February 2020: Deadline for Proseminarschein paper (23:55).

31. March 2020: Deadline for Hausarbeitschein paper (23:55).

3 Course Details

3.1 Materials

Course textbook: Linnebo, Øystein (2017) *Philosophy of Mathematics*. Princeton University Press.

Selected papers and book chapters: Available either from the publisher, through JSTOR, and/or ILIAS.

Lecture notes: Available immediately after lectures on ILIAS.

3.2 Lecture Schedule

Week 1 *23. Okt.* Introduction and outline of the course

Week 2. *30. Okt.* Plato and the Pre-Socratics

Week 3. *6. Nov.* Potential Infinity: Aristotle

Week 4. *13. Nov.* The Calculus: Ghosts of Departed Quantities?

Week 5. *20. Nov.* Logicism: Frege

Week 6. *27. Nov.* Logicism: Russell and Whitehead

Week 7. *4. Dez.* Neo-Logicism

Week 8. *11. Dez.* Entfällt.

Week 9. *18. Dez.* The Cantorian Infinite

21. Dezember – 5. Januar 2019: No lectures.

Week 10. *8. Jan.* Intuitionism: Brouwer

Week 11. *15. Jan.* Intuitionism and Meaning: Dummett

Week 12. *22. Jan.* Formalism and Hilbert's Programme

Week 13. *29. Jan.* The Limitative Results I: Gödel and Tarski

Week 14. *5. Feb.* The Limitative Results II: The Halting Problem

Week 15. *12. Feb.* Where Forward? The Continuum Hypothesis and our choice of axioms

3.3 Possible Essay Questions

1. Does Plato's theory of Forms imply that there are infinitely many natural numbers?
2. Is the infinite potential or actual for Aristotle?
3. Is talk of infinity meaningful even if there are no actual infinities?
4. Does Newton and Leibniz's use of infinitesimals show that it is acceptable to use inconsistent mathematical theories?
5. Does the existence of mind-independent abstract mathematical objects imply that every sentence of mathematics is either true or false (and not both)? Does the answer differ across subjects?

6. Is mathematics and our talk of infinity really just part of logic?
7. Should our logic concerning infinity be classical or intuitionistic?
8. Explain and critically assess Cantor's distinction between consistent and inconsistent multiplicities.
9. What is the halting problem? Outline one way in which it is important.
10. Given Gödel's Incompleteness Theorems, was Hilbert's Programme a waste of time?
11. Does the Continuum Hypothesis have a truth value?
12. N.B. I will accept (and indeed encourage!) coming up with your own essay questions. However, you **must** have cleared your question with me at least 2 weeks in advance of essay submission.

3.4 Readings for each week

The course is based around a cluster of issues facing our philosophical understanding of mathematics, and in particular our understanding of infinity. This includes an analysis of the historical development of philosophical attitudes to mathematics, as well as some themes that have arisen more recently.

I have marked the core reading for each week with an asterisk (this is the reading that you accomplish before class). I will post the core reading on ILIAS. Other readings are optional—you are certainly not required to read everything for every week, but if you wish to write a paper on the topic the optional readings will give you additional background, should you want it, but it is not essential to read everything to get a certain grade (I'll be assessing based on the quality of material you write, not the citations you provide). For the extra keen, further references are available in the course notes that I will make available on ILIAS after class each week.

Week 1 *23. Okt.* Introduction and outline of the course.

- (∗) This course outline.

Week 2. *30. Okt.* Plato and the Pre-Socratics

- (∗) Plato, *Republic* 514–534d (or the whole of Book VII, for the keen). Widely available online, but I like the translation in the 1997 Cooper-edited *Hackett* version of the *Complete Works*.
- (∗) Couprie, Dirk (2019) 'Anaximander', §§1–4. Available from the *Internet Encyclopaedia of Philosophy*: `https://www.iep.utm.edu/anaximan/#H4`. First four sections provide an overview of some of Anaximander's ideas, as well as the surviving fragment.
- Moore, A.W. (1990) *The Infinite*, Chapter 1 'Early Greek Thought'. Routledge. Provides a general survey of the ideas from Plato and earlier.
- Annas, Julia *Aristotle's Metaphysics Books M and N*, 'Introduction' §1. Oxford University Press, 1976.

Week 3. *6. Nov.* Potential Infinity: Aristotle.

- (∗) Aristotle, *Physics*, Book III (esp. Part 4). Widely available online.
- Moore, A.W. (1990) *The Infinite*, Chapter 2 'Aristotle'. Routledge.
- Annas, Julia (1976) *Aristotle's Metaphysics Books M and N*, 'Introduction' §2. Oxford University Press.

Week 4. *13. Nov.* The Calculus: Ghosts of Departed Quantities?

- (∗) Smith, Sheldon R. (2015) 'Incomplete Understanding of Concepts: The Case of the Derivative'. *Mind* 124 (496):1163-1199. Just read Section 2 (unless you are super-keen) which provides a clear and concise history of the use of the calculus. The paper as a whole is great, but gets into some finer details concerning concepts of derivative.
- (∗) Berkeley, George (1734) *The Analyst*, §§I–XX, L. Widely available online (including a LaTeX-typeset version here: `https://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.pdf`)
- Moore, A.W. (1990) *The Infinite*, Chapter 4 'The Calculus'. Routledge.
- Colyvan, Mark (2008) 'Who's Afraid of Inconsistent Mathematics?; *ProtoSociology* 25:24-35. I don't agree with the arguments in this paper, but I do think that diagnosing why they

are flawed is a useful exercise. You can get it from Colyvan's website here: `http://www.colyvan.com/papers/waoim.pdf`

Week 5. *20. Nov.* Logicism: Frege

- (∗) Linnebo, Øystein (2017) *Philosophy of Mathematics*, Ch. 2 'Frege's Logicism'. Princeton.
- Frege, Gottlob (1884) *Die Grundlagen der Arithmetik*, §§1–4, §§45–91, §§106–109. (I use the J.L. Austin translation, but I imagine it won't be necessary for many members of the class.)
- Giaquinto, Marcus (2002) *The Search for Certainty* Part II, Ch. 3 'Frege's Logicism and his Response to Russell's Paradox'. Oxford University Press.

Week 6. *27. Nov.* Logicism: Russell and Whitehead

- (∗) Giaquinto, Marcus (2002) *The Search for Certainty* Part II; Ch. 1, 3, 4, Part III; Ch. 1, Oxford University Press.

Week 7. *4. Dez.* Neo-Logicism

- (∗) Linnebo, Øystein (2017) *Philosophy of Mathematics*, Ch. 9 'Abstraction Reconsidered'. Princeton.
- Tennant, Neil (2017) 'Logicism and Neologicism' §§2–3. *Stanford Encyclopaedia of Philosophy*. Available here: `https://plato.stanford.edu/entries/logicism/`

Week 8. *11. Dez.* Entfällt.

Week 9. *18. Dez.* The Cantorian Infinite

- (∗) Jané, Ignacio (1995). 'The role of the absolute infinite in Cantor's conception of set'. *Erkenntnis* 42 (3): pp. 375 - 402.
- VSauce (2016), 'How to count past infinity'. An informal YouTube introduction to the Cantorian infinite, available here: `https://www.youtube.com/watch?v=SrU9YDoXE88&t=235s`.
- Moore, A.W. (1990) *The Infinite*, Ch. 8 'The Mathematics of the Infinite, and the Impact of Cantor'. Routledge.

21. Dezember – 5. Januar 2019: No lectures.

Week 10. *8. Jan.* Intuitionism: Brouwer

- (∗) Brouwer, L. E. J. (1913) 'Intuitionism and Formalism'. Bull. Amer. Math. Soc., 20(2). Available online through project Euclid.
- Linnebo, Øystein (2017) *Philosophy of Mathematics*, Ch. 5 'Intuitionism'. Princeton.

Week 11. *15. Jan.* Intuitionism and Meaning: Dummett

- (∗) Dummett, Michael (1975) 'The Philosophical Basis of Intuitionistic Logic', *Studies in Logic and the Foundations of Mathematics*, Vol. 80. Available through Science Direct.
- Dummett, Michael (1977) *Elements of Intuitionism*, Ch. 7 'Concluding Philosophical Remarks' (§7.4 and 7.5 can be ignored unless you are especially keen.). Oxford University Press.

Week 12. *22. Jan.* Formalism and Hilbert's Programme

- (∗) Hilbert, David (1984) 'On the infinite'. In P. Benacerraf and H. Putnam (Eds.), *Philosophy of Mathematics: Selected Readings* (pp. 183-201). Cambridge: Cambridge University Press. Available online here: `https://math.dartmouth.edu/~matc/Readers/HowManyAngels/Philosophy/Philosophy.html`.
- Giaquinto, Marcus (2002) *The Search for Certainty* Part IV; Chs. 3, 4. Oxford University Press.
- Zach, Richard (2019) 'Hilbert's Program', *Stanford Encyclopaedia of Philosophy*. §§1–3. Available here: `https://plato.stanford.edu/entries/hilbert-program/`.
- Linnebo, Øystein, *Philosophy of Mathematics* (2017) Ch. 4, 'Hilbert's Program'. Princeton.

Week 13. *29. Jan.* The Limitative Results 1: Gödel and Tarski

- (∗) Giaquinto, Marcus (2002) *The Search for Certainty*, Part V. Oxford University Press.
- Zach, Richard (2019) 'Hilbert's Programme', *Stanford Encyclopaedia of Philosophy*. §§4–5. Available here: `https://plato.stanford.edu/entries/hilbert-program/`

Week 14. *5. Feb.* The Limitative Results 2: The Halting Problem

- (∗) Penrose, Roger (1989) *The Emperor's New Mind*, Ch. 2 'Algorithms and Turing Machines'. Oxford University Press.

- Hopcroft, John (1984). 'Turing Machines'. *Scientific American*, 250(5), 86-E9.
- Pullum, Geoffrey K. (2000) 'Scooping the Loop Snooper', *Mathematics Magazine*, October 2000, pp. 319-320. An improved version is available here: `http://www.lel.ed.ac.uk/~gpullum/loopsnoop.html`

Week 15. *12. Feb.* Where forward? The Continuum Hypothesis and our choice of axioms.

- (∗) Linnebo, Øystein (2017) *Philosophy of Mathematics*, Ch. 12 'The Quest for New Axioms'. Princeton.
- Maddy, Penelope (1988) 'Believing the Axioms I' *Journal of Symbolic Logic*, 53(2):pp. 481-511.