# Review: Elaine Landry, ed. *Categories for the Working Philosopher*. Oxford University Press, 2017 . ISBN 978-0-19-874899-1 (hbk); 978-0-19-106582-8 (e-book). Pp. xiv + 471

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August 27, 2019†

## Introduction

Saunder's Mac Lane's *Categories for the Working Mathematician* is widely regarded as one of the seminal textbooks in category theory. There he shows how categorial constructions appear in various guises throughout mathematics. This is the pedagogical style of [Mac Lane, 1971]; he explains various categorial properties by providing many examples using commonplace mathematical structures, thereby providing a more concrete understanding of a very abstract subject-matter for the working mathematician. Elaine Landry has collected together several of the leading category theorists and philosophers to try and do similarly for philosophers, explaining category theory by showing how it appears in diverse philosophical contexts. Indeed, she explicitly sets herself this task (along with her collaborators):

> "Borrowing from the title of Saunders Mac Lane's seminal work *Categories for the Working Mathematician*, this book aims to bring the concepts of category theory to philosophers working in areas ranging from mathematics to proof theory to computer science to ontology, from physics to biology to cognition, from mathematical modeling to the structure of scientific theories to the structure of the world." (Preface, p. vii)

Not content with this, she adds the following target:

> "Moreover, it aims to do this in a way that is accessible to a general audience. Each chapter is written by either a category-theorist or a philosopher working in one of the represented areas, and in a way that is accessible and is intended to build on the concepts already familiar to philosophers working in these areas." (Preface, p. vii)

Given the title of the work, and Landry's description in the Preface, one might be tempted to think that with *Categories for the Working Philosopher* we have a new textbook on category theory aimed at philosophers, and one that is especially accessible and shows how category theory is directly applicable in philosophy.

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†I am grateful to Chris Scambler for some comments on a draft of this review, as well as the support of the VolkswagenStiftung through the project *Forcing: Conceptual Change in the Foundations of Mathematics*.

This is not that book. This said, I am nonetheless enthusiastic about the work, and would recommend it to anyone who has finished an introductory textbook in category theory, or more generally anyone interested in applications of category theory in philosophy (providing that they go in with the mindset of not getting too bogged down in the details). Perhaps this review can help potential readers navigate the tricky terrain. All in all, Landry has done a fine job of bringing together some of the best minds in category theory and its philosophy on a timely topic that is ripe for further discussion.

In what follows I'll first very briefly summarise the chapters. I'll then give an overall appraisal of the structure and purpose of the work. I won't, however, engage in too much critical analysis of the individual essays; with 18 pieces by 20 different authors, any such analysis would inevitably be a little superficial. Throughout, names in square brackets refer to the author's contribution to *Categories for the Working Philosopher*; the full list of essays with pagination can be found at [*Philosophia Mathematica*, 2018].

# 1 The Essays

Landry helpfully summarises the chapters of *Categories for the Working Philosopher* in the Preface, and divides the book into two parts. Roughly speaking, chapters 1–10 are 'pure' in the sense that they deal with issues central to logic and mathematics. Chapters 11–18, on the other hand, deal with applications of category theory to philosophical debates surrounding topics such as quantum mechanics, theories of space-time, biological organisms, and the subject-matter of scientific theories.

## 1.1 Pure

Colin McLarty's essay 'The Roles of Set Theories in Mathematics' argues that whilst the "orthodox set theory" underlying mathematics is commonly taken to comprise **ZFC**, this is not supported by actual discourse in (non-set-theoretic) texts on mathematics. Instead, he argues that **ETCS** (a categorial theory of sets) can do the job better. Pleasingly, he does the hard work of finding texts in mathematics that support his claim. Occasionally it feels as though some of the positions he is attacking are something of a straw-man; for example he often criticises set theorists for thinking that numbers are *literally* von Neumann ordinals. To my mind, it is at least open whether or not material[1] set theorists ascribe to such a strong set-theoretic reduction (e.g. by thinkng that the number 2 is *literally* $\{\emptyset, \{\emptyset\}\}$). Instead many opt for the von Neumann interpretation of natural numbers as a device of *representation*, a claim evidenced by the fact that set theorists will happily assume that ordinals are represented by restricted equivalence classes when constructing an ultrapower embedding from a measurable cardinal. Despite this mild quibble, the points addressing what most mathematicians really *need* from an underlying set theory (e.g. that functional composition be associative and have identities, or that functions are determined by their effects on elements) are important and worth further scrutiny.

David Corfield's 'Reviving the Philosophy of Geometry' begins by stating that the philosophy of geometry has largely been neglected in the Anglophone community at least outside its uses in physics. While I do not agree with the first claim

[1] *Material* set theories are those which involve axiomatising a single non-logical symbol '$\in$'. They are to be contrasted with *categorial* or *structural* set theories that aim to characterise in categorial terms the topos **Set**, and in which membership is a symbol defined in terms of arrows from terminal objects.

when read in full generality[2], much of the philosophical literature on geometry is focussed on traditional questions concerning the epistemology and metaphysics of the Euclidean plane, whereas Corfield has more recent mathematical developments in mind. Corfield goes on to make some interesting points, but his main focus is that modern geometry is best understood through Homotopy Type Theory[3] (or HoTT); a framework with close connections to category theory.

Michael Shulman's 'Homotopy Type Theory: A Synthetic Approach to Higher Equalities' provides a survey of some aspects of the Homotopy Type Theory and Univalent Foundations[4] programme (including $\infty$-groupoids). It represents a welcome intuitive summary and primer for those wanting to approach the more detailed [The Univalent Foundations Program, 2013]. Shulman also provides a clear and concise example of an application, namely to general covariance in the study of space-time manifolds.

Steve Awodey's 'Structure, Invariance, and Univalence' continues the HoTT-theme by considering how the Univalence Axiom might be supported by structuralist considerations. He argues that we should define structure via an abstraction principle and isomorphism, where isomorphisms are understood category-theoretically[5]. Using HoTT with the Univalence Axiom added we then arrive at a theory with a high degree of structural invariance.

Michael Ernst's 'Category Theory and Foundations' surveys some categorial foundations (in particular $\mathbf{ETCS}$ and $\mathbf{CCAF}$)[6] and their features. He also surveys some of the criticisms of foundational adequacy levelled at both kinds of foundation, in particular that $\mathbf{ETCS}$ is too weak to carry out all desired constructions in analysis ([Mathias, 2000], [Mathias, 2001]), and various kinds of criticism levelled against material set-theoretic foundations (e.g. that it always misses some mathematical subject matter when interpreting large categories either through the use of Grothendieck universes or class theory). He then considers [Feferman, 2013]'s desiderata on a foundation for category theory, one should be able to: (1.) form the category of all structures of a given kind, e.g. the category of all groups, topological spaces etc. (2.) form the category of all functors from $A$ to $B$, where $A$ and $B$ are any two categories, and (3.) establish the existence of the natural numbers, and perform various kinds of mathematical construction (e.g. taking unions, products, etc.). He then explains his own result (in [Ernst, 2015]) that (1.)–(3.) are not jointly satisfiable, before discussing the significance of this fact for categorial foundations.

Jean-Pierre Marquis' 'Canonical Maps' argues that within mathematics there are certain mappings that are 'canonical' in the sense of being uniquely determined by some data (or part of that data). Category theory, he argues, highlights and provides examples of certain mathematical mappings as privileged, for example when the

[2] The first five chapters of [Giaquinto, 2007], and the references contained therein, is a good starting point for seeing that the philosophy of geometry is alive and well.

[3] For the uninitiated: Homotopy Type Theory (or HoTT) comprises comprises a cluster of frameworks in which intuitionistic type theories are endowed with homotopical interpretations. It's a rich area of contemporary study, with some interesting possible implications with respect to the formalisation of mathematics (notably with respect to computerised proof-assistants). See [The Univalent Foundations Program, 2013] for a textbook treatment.

[4] Univalent Foundations is Homotopy Type Theory with the Univalence Axiom added (which, roughly speaking, says that the identity relation is (in a certain precise sense related to isomorphism) equivalent to equivalence).

[5] The category-theoretic definition of isomorphism is that $A$ is isomorphic to $B$ iff there are maps $f : A \to B$ and $g : B \to A$ such that $f \circ g = Id_B$ and $g \circ f = Id_A$. It differs from the set-theoretic definition in that there are category-theoretic isomorphisms that are not underwritten by bijections. See [Marquis, 2013] for some philosophical discussion.

[6] $\mathbf{ETCS}$ is an attempt in [Lawvere, 1964] to axiomatise the category of sets, $\mathbf{CCAF}$ attempts to axiomatise the category of all categories (see [Lawvere, 1966]).

relevant morphism is the unique one with a certain property within a category.[7] He gives several examples of contexts in which one would assert that a particular map is canonical, and then argues that these canonical maps provide a unified class of global organising principles of mathematics (what he calls the "archetechtonic" of mathematics).

John L. Bell's 'Categorial Logic and Model Theory' provides a handbook-style exposition of how one can do semantics using topoi[8]. After some enlightening historical remarks, he explains how a the algebra of subobjects in a category forms a Heyting algebra, and hence that the internal logic of a topos is in general intuitionistic (though it may be classical). He also links this to the internal language of a topos, and provides exposition of the fact that every topos determines a particular (intuitionistic) type theory, and every intuitionistic type theory generates a topos. He then describes functorial semantics with quantifiers interpreted as adjoints, how one can do model theory categorially, and explains geometric theories[9] and their categorial properties.

Jean-Pierre Marquis' 'Unfolding FOLDS: A Foundational Framework for Abstract Mathematical Concepts'[10] explains how Makkai's theory of **F**irst-**O**rder **L**ogic with **D**ependent **S**orts (**FOLDS**) can be used as a language that is invariant with respect to certain abstract concepts. In particular he gives an exposition of **FOLDS**, before describing an invariance theorem that, for two models $\mathfrak{M}$ and $\mathfrak{N}$, and a certain kind of equivalence $\equiv_{\mathcal{L}}$, $\mathfrak{M} \models \phi$ and $\mathfrak{M} \equiv_{\mathcal{L}} \mathfrak{N}$ implies that $\mathfrak{N} \models \phi$, thereby showing that **FOLDS** respects invariance. He then outlines some details concerning higher-dimensional categories, before reviewing some philosophical upshots (e.g. that **FOLDS** only expresses invariant properties, and hence is of interest for structuralism).

Kohei Kishida's 'Categories and Modalities' presents a detailed explanation of how modal logic and category theory are often related. He exposits some categorial relationships within propositional logic, Kripke semantics, and connections between topology and modal logic, as well as a categorial study of quantification and free logic. He then provides discussion of some philosophical questions (e.g. regarding the use of epistemic modality, impossible worlds, and counterpart theory). He also interestingly argues that the idea that the Converse Barcan Formula is solely about increasing domains is somewhat misleading; the Converse Barcan Formula holds more generally in non-Kripkean semantics when the domains are autonomous (in a certain precise sense). This condition of autonomy then corresponds to increasing domains in the Kripkean framework.

J. R. B. Cockett and R. A. G. Seely's 'Proof Theory of the Cut Rule' shows how the Cut Rule can be given a categorial semantics in terms of multicategories and polycategories. For example, in certain contexts Cut corresponds to categorial composition. Moreover, they provide a detailed explanation of how *circuits* are related to the proof theory of fragments of linear logic, in particular providing a nice visualisation (with formal backing) of several notions including commutativity. Using this they are then able to provide an interesting treatment of negation as a contravariant functor, and

[7]This state of affairs is often referred to as a *universal property*.

[8]Topoi (or toposes) are particular kinds of category that contain the right kinds of morphism to admit of an internal logic. They can be viewed as a generalisation of the category of sets, but where one object being a subobject of another (a generalisation of the idea of subset) can take values in a Heyting algebra, rather than the Boolean algebra 2.

[9]A (finitary) geometric formula of some language $\mathscr{L}$ is one which does not contain implication or universal quantification. We then define a a geometric implication as a sentence of the form $\forall x\phi \rightarrow \psi$ where $\phi$ and $\psi$ are geometric formulas. A *geometric theory* consists solely of geometric implications.

[10]Since Marquis has contributed two essays to the volume, we will refer to 'Canonical Maps' as [Marquis-a] and 'Unfolding FOLDS' as [Marquis-b].

present some similarities between their analysis and modal logic. Especially pleasing in this piece is the presentation of a wide variety of visual diagrams to underpin what is quite complex material.

## 1.2 Applied

Samson Ambramsky's 'Contextuality: At the Borders of Paradox' shows how the broad phenomenon of contextuality in quantum mechanics—that we can have locally consistent but globally inconsistent data[11]—is a notion that is both fine-grained (in that there are multiple different strengths of contextuality) and widely applicable. In particular, if we have a set of contexts of observation and a set of features present in these contexts, we can think of the idea that there is a set of possible data descriptions which can arise from performing measurements via a functor. Several interesting results then follow; local consistency corresponds to being a compatible family in a sheaf-theoretic sense, and global consistency is characterised via the usual gluing condition. Thinking of empirical models as families of (certain kinds of) compatible distributions, an empirical scenario is contextual (i.e. locally but not globally consistent) iff it has no global section. He then goes on to examine the Hardy paradox in this light and then compare some more fine-grained notions of contextuality for these empirical models (probabilistically, possibilitically, and strongly contextual). Especially interesting for philosophers concerned with the philosophy of logic more widely is that some of these phenomena turn out to have bearing on technical results like Liar cycles and the Robinson Joint Consistency Theorem. Thus Abramsky shows how category theory can provide useful geometric intuitions (in this case, via the use of bundles) to multiple (often quite abstract) areas.

Bob Coecke and Aleks Kissinger's 'Categorial Quantum Mechanics I: Causal Quantum Processes' uses circuits and diagrams to present a survey of the initial steps in the categorial interpretation of quantum mechanics. They take the interesting approach of taking diagrams composed of boxes and wires (rather than category theory) as their starting point, and then use them as a representation for categorial properties. They view processes as diagrams, and wiring together of processes as a way of generating new processes. This can then be used to describe a process theory applicable to many areas (e.g. physics, chemistry, biology, computation), but the focus of their paper is quantum mechanics. Using their diagrams, they define different diagrammatic notions with distinct purposes, that can be put to use in interpreting certain kinds of physical structure. In particular (i) circuit diagrams (that admit causal structure) correspond to symmetric monoidial categories, (ii) diagrams (where outputs are always wired to inputs) correspond to traced symmetric monoidial categories, and (iii) string diagrams (where inputs can also be wired to inputs and outputs to outputs) are related to compact closed categories. Using these diagrams they provide a discussion of quantum teleportation, quantum types, and give an analysis of causality in this setting. As with many of the papers in the volume, it is very good at situating the abstract material with concrete examples from fields such as computing and regular-everyday-life, and provide helpful diagrams to aid visual intuition (as well as some that contain enjoyable comic relief).

James Owen Weathrall's 'Category Theory and the Foundations of Classical Space-Time Theories' surveys some notions of sameness and difference in structure between space-time theories using categorial tools. He begins with a helpful introduction to forgetful functors. He explains how functors can forget (i) structure when

[11] Examples of contextuality include the Kochen-Specker Paradox, Bell's Theorem, and the Hardy Paradox.

they are not full (as when we take a functor $F$ : Top $\rightarrow$ Set forgetting topological structure), (ii) properties when they are not essentially surjective (for example $F$ : AblGrp $\rightarrow$ Grp from the category of Abelian groups to the category of groups forgets the property of being Abelian), and (iii) *stuff* when they are not *faithful* (e.g. the functor $F$ : Set $\rightarrow 1$, where $1$ is the category consisting of a single object and single identity arrow, forgets all the elements of sets). He then uses these notions of forgetfulness to analyse how one theory can have more structure than another. For example, it is an observation of Earman that Newtonian space-time has more structure than Galilean space-time; in the former we have a notion of absolute rest (allowing us to define absolute velocity) whereas in the latter we do not. Thus, we have more symmetries when working in the Galilean framework. This is brought out by considering the category Gal and New which have Galilean and Newtonian space-time as their objects, and automorphisms thereof as their arrows. There is then a functor $F$ : New $\rightarrow$ Gal that is essentially surjective and faithful, but not full, and so forgets only structure. He shows that there is a similar functor forgetting structure between the category of models of Newtonian Gravitation (NG) and Geometrised Newtonian Gravitation (GNG), and categories corresponding to the theory of electromagnatism formulated with vector potential ($EM_2$) and the version formulated using the Faraday tensor ($EM_1$). However, in this latter case, adding in the gauge transformations as extra arrows in $EM_1$ allows us to have a functor to the category corresponding to $EM_2$ forgetting nothing. He then uses this idea of analysing an excess of structure through forgetful functors to examine other gauge theories, in particular Yang-Mills theory and general relativity. Specifically, by defining a category from Yang-Mills theory, Weatherall surveys results that show that there is a functor from this category to the category associated with $EM_1$ which does not forget structure. He then goes on to discuss a result that there is a functor (forgetting nothing) from the category of relativistic space-times (where the objects are relativistic space-times and arrows are isometries) to the category of Einstein algebras (with objects Einstein algebras and arrows homomorphisms preserving the relevant metric). This has immediate bearing on Earman's claim[12] that formulating general relativity in terms of Einstein algebras results in a theory with less excess structure than the standard formalism.

Joachim Lambek's posthumous 'Six-Dimensional Lorentz Category' presents a view on which time is understood as two-dimensional. Specifically he argues that mathematical elegance would require three dimensions of time, but these can be reduced to two via a categorial proof that the first-order Dirac equation is equivalent to the second-order Klein-Gordon equation. We can then consider a finite additive category with three objects, whose arrows correspond to four-vectors, six-vectors, and Dirac spinors of four-dimensional relativistic quantum mechanics. Lambek then represents six-dimensional space-time using quaternions, showing that (in a certain sense) the three timelike dimensions can be reduced to two. He then applies this to show that certain elements of relativistic quantum mechanics can then be represented, before closing with some brief remarks on the possible physical meaning of the two dimensions.

Andrée Ehresmann's 'Applications of Categories to Biology and Cognition' provides an explanation of how category theory can be used to systemise relationships in dynamic systems; structures like organisms that might change their parts and internal organisation through time. This representation is achieved by having a categories indexed by points on some timescale, and certain functors indicating the possible transitions from one state to another (these turn out to be semi-sheaves of categories). Hierarchical systems of biological structures (e.g. as in the relationship

[12] See [Rosenstock et al., 2015] for the details.

of cell to molecule, and organ to cell) can then be represented by further indexing and colimits. For these hierarchical systems, complexity can be seen to be part of their ability to have components playing radically different roles, something which is formalised by the *Multiplicity Principle*; a requirement on these systems having decompositions which are independent in a certain precise category-theoretic sense. Ehresmann then shows that this property is preserved by *complexification processes*; categorial interpretations of the selection of particular successive states for the hierarchical system. She then uses this to further systematise relationships between several biological concepts including the memory of an organism, before providing an application of these concepts and frameworks to neural and mental systems.

David I. Spivak's 'Categories as Mathematical Models' argues that various kinds of mathematical models of intuitive ideas (e.g. linearity, symmetry) are best understood via their relationship to other models (an idea naturally formalised in category theory). In this way, we can think of category theory as a *universal modelling language*. He begins by noting that category theory leaves the specific encoding of mathematical properties entirely absent, allowing one to categorise the hierarchical nature of models rather than getting bogged down in coding questions. He makes some philosophical remarks about the nature of modelling to the effect that we wish to emphasise certain observable aspects of an environment. He then discusses the idea that the value of a model lies in its interactions with other models, an idea which he relates to the work of Kant. Next, he examines various algebraic structures including; the group of invertible $n \times n$ Matrix multiplication, the monoid of $n \times n$ matrix multiplication, the category of matrix multiplication, and the group-enriched category of matrix arithmetic. He then considers vector spaces as models of *linearity*, and then argues that higher-order vector spaces are models of *linearity itself*. Similarly groups can be understood as models of *symmetry*, whereas the category of all groups is a model of *symmetry itself*. He then considers some applications for this framework, before closing with some final remarks about how category theory is not to be viewed as a foundational language in which science should be done, but rather as a tool to yield conceptual clarity, if so desired.

Hans Halvorson and Dimitris Tsementzis' 'Categories of Scientific Theories' suggests that we can view the structure of all scientific theories as a 2-category of categories. They begin by reviewing two ways we may think of theories as categories. First, we obtain a category from a theory $\mathbf{T}$ *syntactically* by taking as objects formulas in contexts, and as arrows equivalence classes (up to $\mathbf{T}$-provable equivalence) of $\mathbf{T}$-provably functorial relations. Second, we could look at the category of models of $\mathbf{T}$ that has models of $\mathbf{T}$ as objects and either homomorphisms[13] or elementary embeddings as arrows (these are denoted by '$\mathsf{Mod}(\mathbf{T})$' and '$\mathsf{Mod}_e(\mathbf{T})$' respectively). There are then different notions of equivalence we could examine for theories. We could treat two theories $\mathbf{T}$ and $\mathbf{T}'$ as equivalent when $\mathsf{Mod}(\mathbf{T})$ and $\mathsf{Mod}(\mathbf{T}')$ (or possibly $\mathsf{Mod}_e(\mathbf{T})$ and $\mathsf{Mod}_e(\mathbf{T}')$) are equivalent as categories. Alternatively, we could examine *syntactic* categories and look at either standard categorial equivalence or Morita equivalence[14]. Next, they argue that neither of the semantic options ($\mathsf{Mod}(\mathbf{T})$ and $\mathsf{Mod}_e(\mathbf{T})$) is an adequate representation of $\mathbf{T}$. They then consider different candidates for the category of all theories (conceived of as syntactic categories), before opting for $\mathsf{Pretop}$; the category of all pretoposes. Under this suggestion, the category of first-order theories is then a 2-category. They then consider some results that seem

[13] Interestingly Halvorson and Tsementzis use a different notion of homomorphism from the version standard in model theory; it is not required to be one-to-one, and equality need not be respected (in a certain sense).

[14] *Morita equivalence* is an idea from topos theory of two theories having a common definitional extension, in a certain precise sense.

to suggest that there may be a duality between the syntactic theories as conceived of through Pretop and a characterisation of semantic theories through the category of topological groupoids, before closing with some some directions for future research and possible philosophical payoffs for the philosophy of science.

Landry herself writes the final chapter entitled 'Structural Realism and Category Mistakes'. In her essay she targets several positions making ontological claims from formal ones in the philosophy of science, and at the core of her argument is the claim that category mistakes (in the ordinary philosophical sense of the term) are often made when trying to make claims about the structure of the world. Of particular focus is the debate surrounding structural realism (and its ontic and epistemic varietes; the claims that all there *is* is structure, and the claim that all we *know* is structure, respectively) as a way of threading the needle between the no-miracles argument and pessamistic meta-induction. She proposes an *in re* Hilbertian version of structuralism, and contends that the language of category theory can be used as a conceptual tool, enabling one to talk about structure whilst eschewing any mention of objects.[15] Throughout, as well as providing an interesting position, she helpfully and carefully clarifies portions of the debate. A nice example is her identification of three possible roles for category theory in the talking about structural realism: (1.) It can be used as a meta-level formal framework for a structural realist account of the structure of scientific theories, (2.) One might appeal to the category-theoretic structure of a *particular* successful theory in arguing we should be epistemically/ontically committed to this theory, and (3.) One can use category theory to make sense of the claim that it is possible to talk about structure without talking about relata or objects. She uses these distinctions to argue against some other rival positions. For example, she criticises French's arguments for ontic structural realism as using claims about (1.)—the appropriate language for formal frameworks—to make ontological claims (and hence making a kind of category mistake). She takes aim at [Bain, 2013]'s arguments for radical ontic structural realism (that structure exists independently of the objects instantiating it) by arguing that physical motivation at the level of mathematical theory is not the same as object-level physical significance. She then uses support of (3.) to argue that there is a conceptual (though possibly not physical) collapse between radical ontic structural realism and the 'more balanced' version of structural realism explained in [Lam and Wüthrich, 2014].

## 2 Appraisal

The book contains high-quality material, bringing together many talented authors. The breadth of the material covered showcases the fruits of category theory in a diversity of different areas, and the authors have done a good job of keeping difficult material as concisely expressed as possible. Moreover, the book is pleasingly cross-referenced, and it certainly feels as though the work is a collaborative collection of essays, rather than a mere conglomerate of separate works.

I do, however, have a few comments on the shape and structure of the collection. The first is a simple point I already mentioned in the introduction; this is definitively *not* a textbook designed to help experienced philosophers approach category theory (in contrast to what Mac Lane did for mathematicians). Whilst the book does a good job of showing how category theory is applicable in a variety of contexts, in order

[15] It is interesting here that these debates echo ones in the foundations of mathematics as to whether or not category theory presupposes some notion of object and/or collection, see [Feferman, 1977] for the original criticism, and [Linnebo and Pettigrew, 2011] for analysis and a survey of responses.

to really appreciate what is going on in each essay one would have to have a really quite significant background in category theory already (probably at least a first course/textbook is required). At many points in several essays, notions are assumed rather than defined and explained. Whilst this is a necessary part of producing material concise enough to cover such a wide range of topics in a manageable amount of time, it is nonetheless it is likely to confuse the student, even if not the expert.

For this reason I think some more introductory material, early on, with obvious application to philosophy, would have been welcome. Whilst one can piece together introductions to the basic concepts of category theory from various other textbooks (e.g. [Goldblatt, 1984], [McLarty, 1992], [Awodey, 2010]), very little time is spent setting the stage and core definitions (a little occurs in [Bell], but this appears mid-way through the volume). The book is already reasonably long, running to approximately 450 pages, so perhaps considerations of space prevent the inclusion of such material. A short guide to the literature or brief introduction to the core concepts for the uninitiated philosopher would have nonetheless been a handy contribution.[16]

This brings us on to a further point concerning the way the book is structured. Landry explicitly divides the book into two parts, the first being pure and concerning the use of category theory in the philosophy of mathematics and logic, and the second concerning applications (especially in the sciences). However, one might question if this is the most helpful taxonomy, given the structure of the articles and the hoped aim of bringing category theory to philosophers. One might instead opt for the following division of papers into the following three categories: (1.) Survey articles, designed to give a helpful overview of an area but not delving too deeply into the details (e.g. [McLarty], [Shulman], [Ernst], [Coecke and Kissenger], [Weatherall]), (2.) Handbook-style articles, designed to give a thorough but concise explanation of the details of the relevant mathematics (e.g. [Bell], [Marquis-b], [Kishida], [Cockett and Seely], [Abramsky], [Lambek], [Ehresmann], [Spivak]), and (3.) Research articles, making specific arguments about particular aspects of category theory (e.g. [Corfield], [Awodey], [Marquis-a], [Halvorson and Tsementzis], [Landry]). It is possible that organising the papers along these lines rather than by subject-matter would help to introduce the reader more gently; first beginning with some overviews of various areas, then delving into technical details but with a focus on exposition, before finally presenting some new (and often quite complicated) research.

The focus on the difference between pure and applied, rather than pedagogical function, results in some slightly unusual aspects of the order. For example, [McLarty] and [Ernst] are both survey-style articles dealing with the use of category theory as a foundational theory, and how it behaves in contrast to set theory. They are separated, however, by [Corfield] which is a challenging (but interesting) article on geometry and Homotopy Type Theory referencing several difficult constructions. Issues with the ordering like these somewhat disrupt the flow of the book, and similar clashes occur quite frequently. For example [Shulman] occurs after [Corfield], but presents some ideas that would have been helpful for reading the latter, [Cockett and Seely] is separated from [Coecke and Kissenger] by [Abramsky] despite the fact that both are intimately concerned with circuits, and [Weatherall] appears after many other more difficult essays, despite the fact that the category theory contained therein is relatively accessible.

A final point (and I should flag here that I am biased) is that many of the pieces (e.g. [McLarty], [Corfield], [Shulman], [Awodey], [Marquis-a], [Coecke and Kissenger]) situate category theory in direct opposition to set theory as a foundational lan-

[16] I hope that this review can at least provide a partial guide for newcomers.

guage. There are some exceptions here; [Ernst] provides some defence of set theory, but doesn't really analyse how they may be complementary (other than remarks about how the hierarchical structure enforced by contemporary set theory may be useful for facilitating a size distinction necessary for certain parts of category theory[17]), [Marquis-a] does acknowledge that set theory is "combinatorially rich" (p. 106) and in [Marquis-b] he views then as non-competing, however he claims that set theory "does not encode mathematical objects properly" (p. 139). [Bell] is also not anti-set theory, and [Halvorson and Tsementzis] explicitly refrain from trying to adjudicate the debate. Despite these exceptions, many of the essays stand in opposition to set theory. Whilst anti-set-theoretic foundationalism is a legitimate position, one worthy of close scrutiny, my concern here is that certain deep foundational connections may be missed by viewing the two languages as competitors in a winner-takes-all bout to determine the one best foundational framework, or even if they are viewed merely as orthogonal disciplines. This is especially so when one considers the fact that fusions of perspective have yielded some foundational fruits, for example the connections between set-theoretic forcing and sheaves are well-known (this is mentioned, but not dealt with in depth, by [Bell][18]). More recently, Bagaria and Brooke-Taylor have shown that category theory can be used to calibrate various strong large cardinal principles (see here [Bagaria and Brooke-Taylor, 2013]). Whilst these observations possibly lie outside the scope of Landry's collection, it is possible that a philosopher might get the impression that the friend of set-theoretic foundations is locked in battle with her category-theoretic counterpart, where in reality it is at least possible that the two have much to learn from one another.

These comments though, should not detract from the fact that the book contains important and interesting work. Landry has curated some wonderful essays on category theory for the working philosopher, and as long as one goes in with an understanding of the difficulty of the material, I can heartily recommend it to the philosopher wishing to know more about category theory and its applications.[19]

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[17] See here [Shulman, 2008].

[18] See the Appendix to [Bell, 2011] for a clear and concise exposition.

[19] Landry provides a rather heartening anecdote in the Preface, one which sums up the situation quite well:

> "Finally, I end this Preface with a short story that might assist the weary, or even fearful, reader. Years ago, as a graduate student, I attended a category theory conference in Montreal and was sitting beside Saunders Mac Lane. During one of the talks, Saunders looked to me and said: "Are you following all of this?" I replied, rather embarrassed: "No." There was a slight pause (for effect, I'm sure), and Saunders then turned with a grin and said: "Neither am I (another pause) and I invented it!" I share this if only to remind the reader that it's ok not to follow all of it!" (Preface, p. ix)

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