

# Research Proposal: Grades of Pluralism in the Foundations of Mathematics.

Neil Barton

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## 1 Current Research

My current research examines whether mathematical practice constitutes evidence against Universism in Set Theory: the view that there is a single, unique, maximal interpretation of set-theoretic discourse. The debate is important in the Foundations of Mathematics; Set Theory is capable of representing most (if not all) mathematical objects, and Universism provides a natural way of looking at this subject matter.

In the course of my research I have found an extant symbiosis between two views. The Universist can incorporate various insights from its polar opposite: the Multiversist position that there are many different possible interpretations of particular set-theoretic sentences, no one of which is privileged. In particular, the logical resources that I argue the Universist can use in assessing new axioms for truth derives from Multiversist-led philosophy and mathematics. Multiversists have also learnt greatly from mathematics inspired by largely Universist projects (such as work by Woodin on Inner Model Theory). Moreover, *via* interesting technical devices, each view can represent the other within their particular framework. The conclusion that has emerged from work completed during my PhD has been clear; none of the arguments given for or against the two kinds of view are convincing. Indeed, these observations call into question the extent to which there is a fruitful debate concerning ontology to be had; both views represent tenable yet distinctive views of set-theoretic ontology.

## 2 The Proposal

As no good arguments either for or against the two views have been given, we might think that working under a spirit of tolerance in the Foundations of Mathematics is a fertile mode of enquiry.

### 2.1 Scientific Pluralism in Set Theory

To see this more clearly, observe that we may be in one of the following three positions concerning Universism and Multiversism:

1. There is a good argument that decides in favour of either Universism or Multiversism, it is just that we have not discovered it yet.
2. There is a fact of the matter about which of Universism and Multiversism is true, but we can never know.
3. There is no fact of the matter concerning Universism and Multiversism.

While (1.) is epistemically possible, the fact nonetheless remains that since the discovery of diverse models of Set Theory there has been a good deal of attention directed towards finding strong arguments without much success. Given this,

analysing possibilities (2.) and (3.) in a spirit of philosophical tolerance is a worthwhile endeavour.

A natural response to this issue is to adopt a Scientific Pluralism towards Set Theory. There are two senses in which this might be understood:

- (a) We may be *methodological* Scientific Pluralists, in the sense that we think that having scholars working under the ideas of both views is productive, but remain neutral as to the subject matter underlying the discipline.
- (b) Alternatively, we might opt for a *deep* Pluralism, on which we regard Universism and Multiversism as providing a partial characterisation of the subject matter of Set Theory. Neither Universism nor Multiversism is strictly correct, but each is a reasonable expression from a particular perspective (similarly to how a person looking at a (finite) solid cylinder end on might report seeing a disc while a person looking at the solid cylinder from the side might report seeing a rectangle).

The first part of the proposal will be to explore these options in greater depth. In particular, the level to which (2.) and (3.) correspond to (a) and (b) requires detailed examination.

## 2.2 Does the argument generalise?

Having provided a detailed analysis of the prospects for a Scientific Pluralism in the context of Set Theory, a natural question to ask is the extent to which this generalises to similar subject matters. Part of what makes the debate concerning Multiversism and Universism so interesting is that pre-theoretically the Universe of sets seems to be a well-defined mathematical structure. It is only after significant mathematical work and deep understanding of the independence phenomenon that this view is challenged.

This indicates an interesting direction of enquiry. In Analysis we also have a subject matter that seems perfectly determinate until mathematical experience teaches us that there are different ways of representing the continuum. We may understand the continuum classically or intuitionistically, with or without infinitesimals. Further, the continuum is particularly interesting as it is a kind of entity that seems to have immediate relevance to our sense experience. Again, we find ourselves in a similar epistemic position with respect to the continuum as we did with Set Theory:

1. There is a good argument that decides in favour of a particular conception of the continuum, it is just that we have not discovered it yet.
2. There is a fact of the matter about which conception of the continuum is correct, but we can never know.
3. There is no fact of the matter concerning which conception of the continuum is correct.

Again, we might take an attitude of either methodological or deep Pluralism towards these epistemic states. Understanding the relationships here and any associated Pluralism would help the community to understand better the philosophical and mathematical nature of the continuum.

To sum up, we present the following key research questions for the project:

1. Are each of the kinds of Pluralism with respect to Set Theory and the study of the continuum tenable views?

2. How do the views relate to our failure to decide conclusively one way or the other?
3. Are there significant disanalogies between the two cases?
4. How should we understand the rôle of each of these views in our mathematical thinking?
5. Are there significant technical insights to be gained by adopting a tolerance between the theories?

This would represent a valuable contribution to the Foundations of Mathematics and would help to clarify its role in understanding Philosophy more widely.